A. SOLVE $6x^2 - 5x - 4$

1. Tell students to not write anything yet. Ask them to look at intensely, and think carefully about, how to approach solving this quadratic equation.

2. Allow students an accurately timed six minutes to solve the equation showing all stages of their working, using the step-by-step formal conventions used in their IB math classes. Announce that, if time permits, they should continue solving the same equation using multiple methods

GENERATIVE QUESTIONS

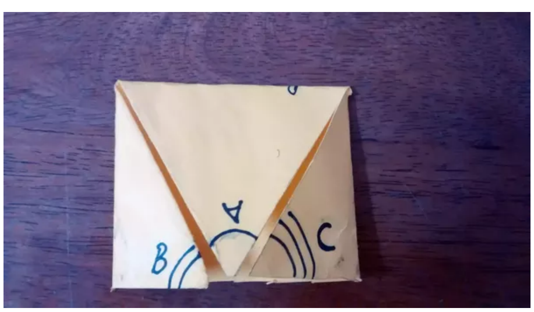
- **If you were an octopus how would you have approached solving the quadratic?**
- **If you were one of the Kindergarteners that you visited in the What do little kids know? unit in Starting TOK, how would you have approached solving the quadratic?**
- **How did *you* approach solving the quadratic?**
- **Try to list the many different ways you can use to solve quadratic equations. Don't worry if you did not get to them all.**

GOING DEEPER

- **To what extent are the various elements of the quadratic equation an extension of the idea of counting numbers?**
- **What is the relationship between algebra and simple arithmetic?**
- **What is the relationship between algebra and geometry?**

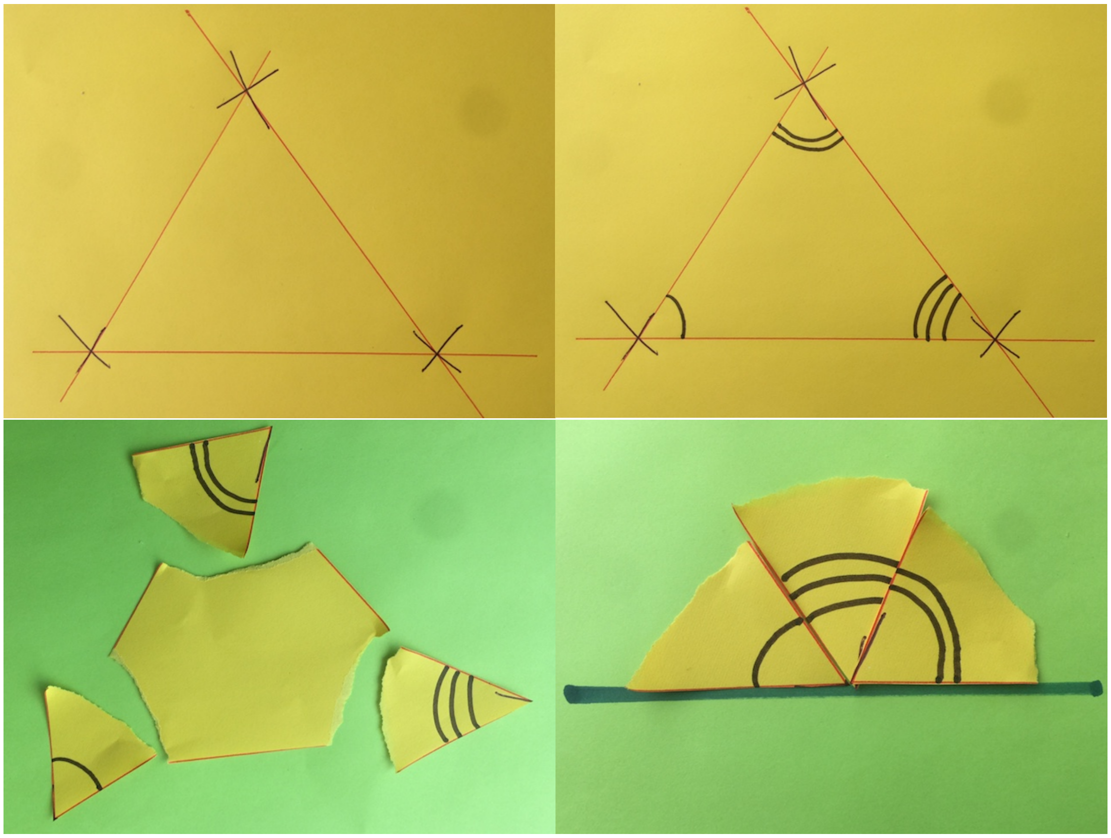
B. SUM OF THE ANGLES IN A TRIANGLE

1. Again tell students to not write anything yet. Ask them to look at intensely, and think carefully about, for a timed 30 seconds, how to reverse engineer the paper model.



2. Tell students to grab a piece of paper quickly, and cut out a random triangle; then they should casually rip off the corners, and arrange the angles, more or less, in a straight line.

3. Next repeat the process in #2 much more systematically by following the four-stage process in these four photographs.



Allow a full 10 minutes for the following tasks. Students are still working individually.

4. Next, return to the geometry you learned in school. Draw a simple geometric diagram with any construction lines you need to prove to your satisfaction visually that the sum of the angles of a triangle add up to 180 degrees.

5. Without using any mathematical symbols write out your proof in full English prose so that your reader would have to visualize your diagram and proof based on only the words you have written on the page (as if it had been written in a novel without the aid of a diagram).

Tell students to sit back-to-back with their partners. Here are the instructions:

- You will read your prose proof from #5. The rule for this task is that your partner can ask you to pause or repeat what you read; but you cannot change what was written. Your partner must sketch or otherwise validate your proof *only* by listening to your text.

- When finished, switch roles. Critique the validity of each other's work.

6. Finally, write out your proof using mathematical symbols as you would in a real math assignment. Use a refined version of your diagram, construction lines, letter symbols and only the barest minimum of elegant prose.

GENERATIVE QUESTIONS

- **Does folding down the corners of a single triangle or ripping off the corners and arranging them in a straight line count as proof that the sum of the angles of a triangle add up to 180 degrees?**

- **Is performing the corner ripping technique more convincing as proof when it is done carefully and accurately, using the ruler to construct the triangle by joining the randomly points marked by crosses?**

- **Would ripping off the corners of hundreds of random single triangles and arranging them in a straight lines count as proof that the sum of the angles of a triangle add up to 180 degrees? ?**

- **A straight line is half a turn in this context; but where does the precise designation of 180 degrees come from?**
- **In terms of rigor and validity, what are the essential differences between the proof techniques you used in #5 and #6?**

- **What do we mean by elegance in proof?**

C. MONTY HALL PROBLEM

GENERATIVE QUESTIONS

- **To what extent do the explanations for the Monty Hall Problem, outlined in the videos, hold up as proof?**

- **In the Monty Hall "hands-on" experiment in the second video, there were twenty repetitions. Notwithstanding a fairly large sample size, the actual results were an exaggerated caricature of the expected results. What was going on?**

- **To what extent are probability and certainty mutually exclusive?**